Journal of Science Engineering Technology and Management Science
Volume 02, Issue 07, July 2025 ISSN: 3049-0952
www.jsetms.com DOI:10.63590/jsetms.2025.v02.i07.146-154

HALL EFFECT ON AN UNSTEADY MHD FREE CONVECTIVE COUETTE FLOW BETWEEN TWO PERMEABLE PLATES

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To Cite this Article

Mr.G.Purushotham, Dr.Maddileti Pasupula, "Hall Effect On An Unsteady Mhd Free Convective Couette Flow Between Two Permeable Plates", Journal of Science Engineering Technology and Management Science, Vol. 02, Issue 07, July 2025,pp:146-154, DOI: http://doi.org/10.63590/jsetms.2025.v02.i07.pp146-154

Submitted: 05-05-2025 Accepted: 12-06-2025 Published: 21-06-2025

Abstract— An investigation on the non – linear problem of the effect of Hall current on the unsteady magnetohydrodynamic free convective Couette flow of incompressible, electrically conducting fluid between two permeable plates is carried out, when a uniform magnetic field is applied transverse to the plate, while the thermal radiation, viscous and Joule's dissipations are taken into account. The fluid is considered to be a gray, absorbing emitting but non – scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The dimensionless governing coupled, non – linear boundary layer partial differential equations are solved by an efficient, accurate, and extensively validated and unconditionally stable finite element method. The effects of thermal radiation and Hall current on primary and secondary velocity, skin friction and rate of heat transfer are analyzed in detail for heating and cooling of the plate by convection currents. Physical interpretations and justifications are rendered for various results obtained.

***Index Terms*— Hall current, Free convection, MHD, Couette flow, Thermal radiation, Finite Element method.**

I. INTRODUCTION

When the strength of the applied magnetic field is sufficiently large, Ohm's law needs to be modified to include Hall current. The Hall Effect is due merely to the sideways magnetic force on the drafting free charges .The electric field has to have a component transverse to the direction of the current density to balance this force. In many works of plasma physics, it is not paid much attention to the effect caused due to Hall current. However, the Hall Effect cannot be completely ignored if the strength of the magnetic field is high and number of density of electrons is small as it is responsible for the change of the flow pattern of an ionized gas. Hall Effect results in a development of an additional potential difference between opposite surfaces of a conductor for which a current is induced perpendicular to both the electric and magnetic field. This current is termed as Hall current. It was discovered in1979 by Edwin Herbert Hall while working on his doctoral degree at the Johns Hopkins University in Baltimore, Maryland, USA. Sivaiah and Srinivasa Raju [1] studied the effects of heat and mass transfer flow with Hall Current, heat source and viscous dissipation using finite element method. Anand Rao et al. [2] demonstrated MHD transient flow past an impulsively started infinite horizontal porous plate in a rotating fluid with Hall current. Ramana Murthy et al. [3] discussed heat and mass transfer effects on natural convective flow past an infinite vertical porous plate with thermal radiation and Hall Current in presence of magnetic field. Hall effect on an unsteady MHD flow past along a porous flat plate with thermal diffusion, diffusion thermo and chemical reaction. Sudhakar et al. [4] studied the effect of hall current on an unsteady magnetohydrodynamic flow past along a porous flat plate with thermal diffusion, diffusion thermo and chemical reaction. Srinivasa Raju et al. [5] discussed heat and mass transfer effects on unsteady flow past an impulsively started infinite vertical plate with hall current using finite element technique. The effect of radiation effects on unsteady MHD free convection with Hall current near on an infinite vertical porous plate studied by Srinivasa Raju et al. [6]. Anand Rao and Srinivasa Raju [7] studied the effect of hall current on an unsteady MHD flow and heat transfer along a porous flat plate with mass transfer and viscous dissipation. Anand Rao and Srinivasa Raju [8] studied the effects of Hall currents, Soret and Dufour on MHD flow and heat transfer along a porous flat plate with mass transfer.

Motivated the above research work, we have proposed in the present paper to investigate the effect of Hall current on the unsteady magnetohydrodynamic free convective Couette flow of incompressible, electrically conducting fluid between two permeable plates is carried out, when a uniform magnetic field is applied transverse to the plate, while the thermal radiation, viscous and Joule's dissipations are taken into account. In section 2, basic equations and dimension less forms of the governing equations are established. Solution method to these equations for the flow variables are briefly examined in section 3. The results of the previous sections are discussed in section 4. In section 5, general concluding remarks of the results of the previous sections are given.

II. BASIC EQUATIONS

An unsteady free convection flow of an electrically conducting, viscous, incompressible fluid past an impulsively started infinite vertical porous plate, in the presence of a transverse magnetic field with the effect of Hall current and thermal radiation are considered. We made the following assumptions.

1. The $x'-$ axis is taken along the infinite vertical porous wall in the upward direction and $y'-$axis normal to the wall.
2. A constant magnetic field of magnitude B_o is applied in $y'-$direction. Since the effect of Hall current gives rise to a force in z' direction, which induces a cross flow in that direction, the flow becomes three dimensional.
3. Let u', v' and w' denote the velocity components in x', y' and z' directions respectively. Let v_o be the constant suction velocity.
4. A uniform magnetic field B_o is applied in the positive $y'-$direction and is assumed undisturbed as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number.
5. It is assumed that the external electric field is zero and the electric field due to the polarization of charges is negligible.
6. The initial temperature of the fluid is the same as that of the fluid, but at time $t' > 0,$ the porous plate starts moving impulsively in its own plane with a constant velocity U_o and its temperature instantaneously rises or falls to T'_w which thereafter is maintained as such.

The governing equations of the problem are as follows:

Momentum Equation:

$$\frac{\partial u'}{\partial t'} - v_o \frac{\partial u'}{\partial y'} = \nu \frac{\partial}{\partial y'}\left(\frac{\partial u'}{\partial y'}\right) + g\beta(T' - T'_\infty) - \frac{\sigma B_o^2 u'}{\rho(1+m^2)}(u' + mw') \quad (1)$$

$$\frac{\partial w'}{\partial t'} - v_o \frac{\partial w'}{\partial y'} = \nu \frac{\partial}{\partial y'}\left(\frac{\partial w'}{\partial y'}\right) - \frac{\sigma B_o^2 u'}{\rho(1+m^2)}(mu' - w') \quad (2)$$

Energy Equation:

$$\rho C_p\left[\frac{\partial T'}{\partial t'}-v_o\frac{\partial T'}{\partial y'}\right]=\kappa\frac{\partial}{\partial y'}\left(\frac{\partial T'}{\partial y'}\right)+\mu\left[\left(\frac{\partial u'}{\partial y'}\right)^2+\left(\frac{\partial w'}{\partial y'}\right)^2\right]$$
$$+\frac{\sigma B_o^2}{\rho(1+m^2)}\left(u'^2+w'^2\right)-\frac{\partial q_r}{\partial y'} \quad (3)$$

The second and third terms on the right – hand side of Eq. (3) represent the viscous and Joule dissipations respectively. We notice that each of these terms has two components. Where $m=\frac{\sigma B_o'}{en_e}$ is the Hall current parameter.

The corresponding boundary conditions of the flow are:

$$\begin{cases} t'\le 0: u'=w'=0,\ T'=T_\infty' \quad for\ all\ y' \\ t'>0: \begin{cases} u'=U_o,\ w'=0,\ T'=T_w'\ at\ \ y'=0 \\ u'\to 0,\ w'\to 0,\ T'\to T_\infty'\ at\ \ y'\to\infty \end{cases} \end{cases} \quad (4)$$

The radiative heat flux term is simplified by making use of the Rosseland approximation as

$$q_r=-\frac{4\bar\sigma}{3k^*}\frac{\partial T'^4}{\partial y'} \quad (5)$$

Here $\bar\sigma$ is Stefan – Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small so that T'^4 can be expressed as a linear function of T' after using Taylor's series to expand T'^4 about the free stream temperature T_∞' and neglecting higher – order terms. This results in the following approximation:

$$T'^4\cong 4T_\infty'^3T'-3T_\infty'^4 \quad (6)$$

Using Eqs. (5) and (6) in the last term of Eq. (3), we obtain:

$$\frac{\partial q_r}{\partial y'}=-\frac{16\bar\sigma T_\infty'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \quad (7)$$

Introducing Eq. (7) in the Eq. (3), the energy equation becomes:

$$\rho C_p\left[\frac{\partial T'}{\partial t'}-v_o\frac{\partial T'}{\partial y'}\right]=\kappa\frac{\partial}{\partial y'}\left(\frac{\partial T'}{\partial y'}\right)+\mu\left[\left(\frac{\partial u'}{\partial y'}\right)^2+\left(\frac{\partial w'}{\partial y'}\right)^2\right]$$
$$+\frac{\sigma B_o^2}{\rho(1+m^2)}\left(u'^2+w'^2\right)+\frac{16\bar\sigma T_\infty'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \quad (8)$$

The physical quantities are cast in the non – dimensional form by using the following dimensionless scheme: $u=\frac{u'}{U_o},\ w=\frac{w'}{U_o},\ y=\frac{y'v_o}{\nu},\ t=\frac{t'v_o^2}{\nu},\ \theta=\frac{T'-T_\infty'}{T_w'-T_\infty'},\ Gr=\frac{\nu g\beta(T_w'-T_\infty')}{v_o^2U_o}$ is the Grashof number for heat transfer, $M=\frac{\sigma B_o'^2\nu}{v_o^2\rho}$ is the Magnetic field (Hartmann number), $\Pr=\frac{\mu C_p}{\kappa}$ is the Prandtl number, $Ec=\frac{U_o^2}{C_p(T_w'-T_\infty')}$ is the Eckert number, $R=\frac{\kappa k^*}{4\sigma T_\infty'^3}$ is the thermal radiation parameter. In terms of the above non – dimensional variables and parameters equations (1), (2) and (8) are, respectively, written as

Momentum Equation:

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)+Gr\theta-\frac{M}{1+m^2}(u+mw) \quad (9)$$

$$\frac{\partial w}{\partial t}-\frac{\partial w}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right)+\frac{M}{1+m^2}(mu-w) \quad (10)$$

Energy Equation:

$$\frac{\partial\theta}{\partial t}-\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\left(\frac{3R+4}{3R}\right)\frac{\partial}{\partial y}\left(\frac{\partial\theta}{\partial y}\right)+\left(Ec\right)\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]$$
$$+\frac{M\left(Ec\right)}{1+m^2}\left(u^2+w^2\right) \tag{11}$$

And the corresponding boundary conditions are

$$\left\{\begin{array}{l} t\le 0: u=w=\theta=0 \quad for\ all\ y \\ t>0:\begin{cases} u=1,\ w=0,\ \theta=1\ at\ \ y=0 \\ u\to 0,\ w\to 0,\ \theta\to 0\ \ as\ \ y\to\infty \end{cases}\end{array}\right\} \tag{12}$$

III. NUMERICAL SOLUTION BY FEM

The numerical technique used to solving the non-dimensional momentum and energy equations (9), (10) and (11) along with the imposed boundary conditions (12) is based on the Galerkin weighted residual method of finite element formulation. Here the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then, the nonlinear governing partial differential equations (i.e. mass, momentum and energy equations) are transferred into a system of integral equations by applying the Galerkin weighted residual method. In this case, the integration over each term of these equations is performed by using Gauss's quadrature method and nonlinear algebraic equations are obtained. These nonlinear algebraic equations are modified by imposing boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. At last, these linear equations are solved by using triangular factorization method.

IV. RESULTS AND DISCUSSIONS

The quantitative differences arise because of different initial and boundary conditions used owing to different physical situations considered. Graphical illustrations of such comparisons are not presented due to paucity of space. It must be noted that negative values of the parameters Gr and Ec correspond to the case of the plate being heated by the convection currents and similarly their positive values correspond to the case of the plate being cooled by the convection currents. We refer to the values of $Gr=\pm 1$ as moderate cooling and heating $Gr=\pm 2$ and as greater cooling and greater heating respectively. In the following discussion, the value of the non – dimensional time is fixed at $t=1.0$. The value of Prandtl number is taken as 0.71 which corresponds to air. Air is assumed to be incompressible since all the velocities considered are less than the velocity of sound in the medium (air) so that the Much number is less than unity. The values for the various parameters are chosen approximately to correspond to a sufficiently ionized air, the flow of which can be modified by an applied magnetic field.

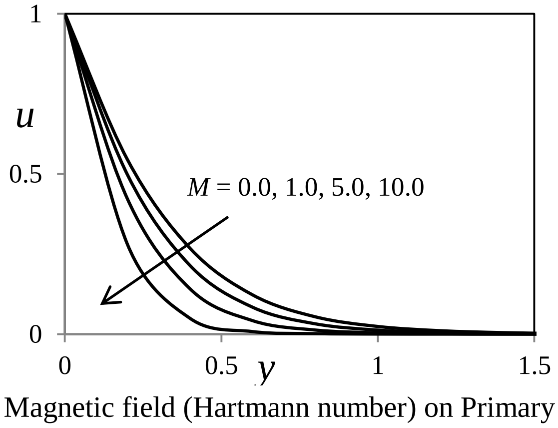

Fig. 1. Effect of Magnetic field (Hartmann number) on Primary velocity profiles

Figures (1) and (2) show that the primary velocity u is diminished and the secondary velocity u is increased due to the applied magnetic field. Indeed, when a transverse magnetic field is applied it is well known that the Lorentz force acts in a direction opposite to the flow and offers resistance to the flow and such a phenomenon is described by the term magnetic – viscosity. From Figs. (3) and (4), it is inferred that the Hall current promotes the flow along the plate, both when the fluid is heated or cooled. This is because, in general, the Hall current reduces the resistance offered by the Lorentz force. It is also observed that the primary velocity u is greater in the case of cooling of the plate than in the case of heating of the plate. Flow reversal is also noticed in the case of greater heating of the plate. The rise and fall in velocity due to cooling and heating of the plate can be explained as follows. In the process of external cooling of the plate, the free convection currents travel away from the plate. As the fluid is also moving with the plate in the upward direction, the convection currents tend to help the velocity to increase. But, in the case of heating of the plate, as the free convection currents are traveling towards the plate, the motion is opposed by these currents and hence there is a decrease in velocity. In the case of greater heating, this opposition is large enough to counteract the upward push offered by the movement of the plate on the fluid particles just outside the thermal boundary layer and the net force acts downwards and hence the flow becomes downwards.

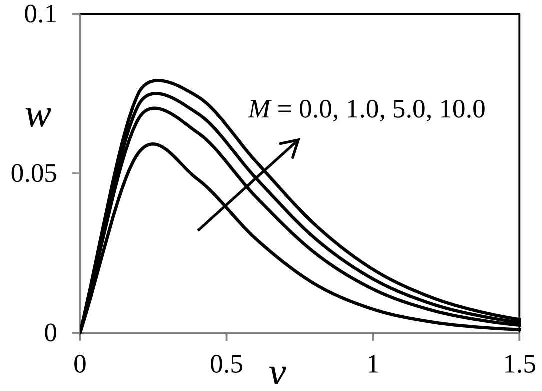

Fig. 2. Effect of Magnetic field (Hartmann number) on Secondary velocity profiles

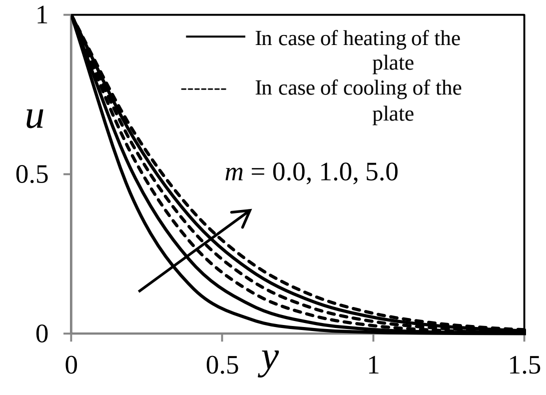

Fig. 3. Effect of Hall Current on the primary velocity profiles for cooling of the plate ($Gr = 1.0$ and $Ec = 0.001$) and heating of the plate ($Gr = -1.0$ and $Ec = -0.001$).

The effect of Hall current on the secondary velocity w is depicted through figs. (5) and (6). The secondary velocity is induced by the component of the Lorentz force in the $z-$direction which arises solely due to the Hall current. From Eq. (11), it is clear that is the term $\frac{Mm}{1+m^2}u$ which decides the flow in the $z-$direction. If the Hall parameter $m = 0$, then the term mentioned above is zero and hence there is no force to induce the flow in the $z-$direction. That is $w=0$. Further, $\frac{m}{1+m^2}$ increases as m increases in the range $0 \le m \le 1$ and it decreases as m increases in the range $m>1$. This means that the magnitude of the component of the Lorentz force in the $z-$direction increases as m increases in the range $0 \le m \le 1$ and hence the secondary velocity w is increased, while it decreases when m increases in the range $m>1$ and hence the secondary velocity w is decreased. These results are observed graphically in Figs. (5) and (6). The secondary velocity w is observed to be greater in the case of cooling of the plate than in the case of heating of the plate. This result also can be inferred from the term $\frac{Mm}{1+m^2}u$ of Eq. (11). The primary velocity u is greater in the case of cooling than in the case of heating of the plate and so is the term mentioned above, which is the deciding factor so far as the secondary velocity is concerned. This ultimately results in the secondary velocity being greater in the case of cooling of the plate than in the case of heating of the plate. By similar arguments, the flow reversal observed in the secondary velocity in the case of greater heating can also be attributed to the flow reversal in the primary velocity.

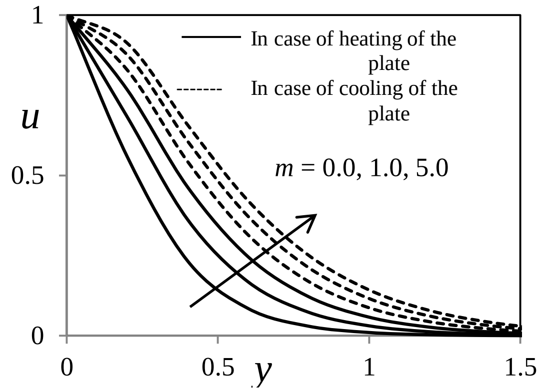

Fig. 4. Effect of Hall Current on the primary velocity profiles for cooling of the plate ($Gr=5.0$ and $Ec=0.003$) and heating of the plate ($Gr=-5.0$ and $Ec=-0.003$).

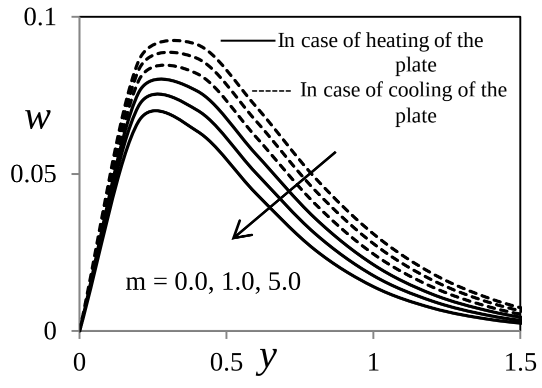

Fig. 5. Effect of Hall Current on the secondary velocity profiles for cooling of the plate ($Gr=1.0$ and $Ec=$ 0.001) and heating of the plate ($Gr=-1.0$ and $Ec=-0.001$).

Figs. (7)-(9) display the effects of the radiation parameter R on the time development of the primary and secondary velocities and temperature of the fluid at the center of the channel. It is observed that increasing R decreases primary and secondary velocities and temperature of the fluid. An increase in the radiation emission, which is represented by R, reduces the rate of heat transfer through the fluid. This accounts for the decrease in temperature with increasing R. The velocity decreases through the reduction in buoyancy forces associated with the decreased temperature.

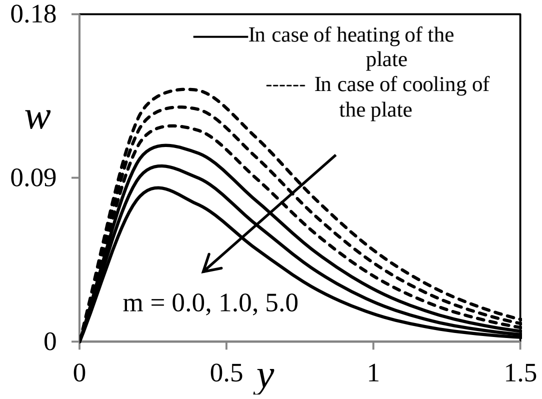

Fig. 6. Effect of Hall Current on the secondary velocity profiles for cooling of the plate ($Gr = 5.0$ and $Ec =$ 0.003) and heating of the plate ($Gr = -5.0$ and $Ec = -0.003$).

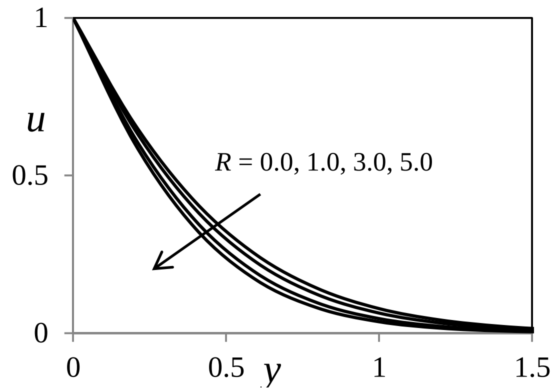

Fig. 7. Effect of thermal radiation on the primary velocity profiles

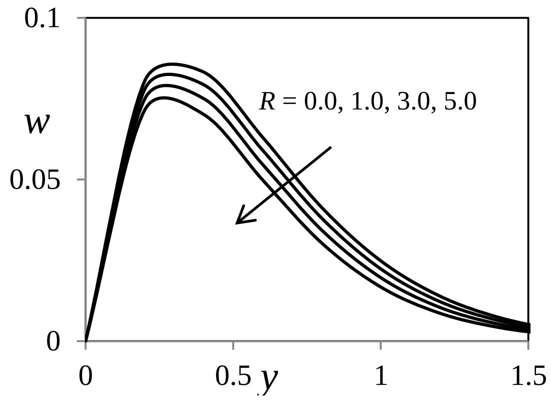

Fig. 8. Effect of thermal radiation on the secondary velocity profiles

The temperature θ is not significantly affected by the magnetic field and Hall current, except in the very close vicinity of the plate. This is because the effect of the Hartmann number and Hall parameter can be felt only in the Hartmann layer and the thermal boundary layer respectively.

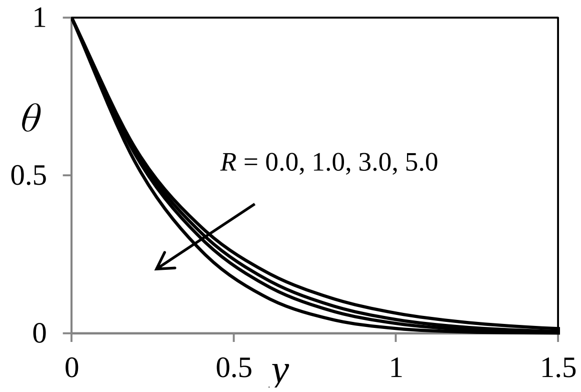

Fig. 9. Effect of thermal radiation on the temperature profiles

V. CONCLUSIONS

The effects of magnetic field, thermal radiation and Hall current on the flow and heat transfer are analyzed and physical interpretations or justifications of the results are provided as and when possible. The results obtained can be summarized as follows.

1. Applied magnetic field retards the primary flow along the plate and supports the secondary flow induced by the Hall current.
2. Hall current promotes the flow along the plate. The secondary flow is supported when the Hall parameter is increased up to unity. If the Hall parameter is increased beyond unity, the secondary flow is retarded. These results are true for both cooling and heating of the plate.
3. Both primary and secondary velocities are found to be greater in the case of cooling of the plate than in the case of heating of the plate. Flow reversal is observed in both primary and secondary velocity components in the case of greater heating of the plate.
4. Magnetic field and Hall current modify only the slope of the temperature profile in the narrow region close to the plate called the thermal boundary layer. Otherwise, their effect on temperature is not significant.
5. Increasing of thermal radiation parameter values, there is a reduction in both primary and secondary velocities. The presence of radiation effects caused reductions in the fluid temperature.

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