MODELING HEAT TRANSPORT IN WALTER'S-B BIO-NANOFLUID FLOW WITH GYROTACTIC MICROORGANISMS

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Submitted: 22-12-2024 *Accepted: 10-01-2025* *Published: 19-01-2025*

ABSTRACT:

The present study investigates heat transport characteristics in the bio-convective flow of a Walter's-B non-Newtonian nanofluid containing gyrotactic microorganisms. The inclusion of nanoparticles enhances thermal conductivity, while gyrotactic microorganisms induce bioconvection, significantly influencing flow stability and heat transfer performance. A mathematical model is developed to describe the coupled momentum, energy, nanoparticle concentration, and microorganism density equations governing the Walter's-B fluid. Appropriate similarity transformations are employed to reduce the governing partial differential equations into a system of nonlinear ordinary differential equations. Numerical solutions are obtained to analyze the effects of key physical parameters such as viscoelasticity, Brownian motion, thermophoresis, bioconvection Rayleigh number, and microorganism concentration on velocity, temperature, and density profiles. The results demonstrate that bio-convection enhances thermal transport by improving nanoparticle distribution and suppressing sedimentation effects. The study provides useful insights into controlling heat transfer in non-Newtonian bio-nanofluid systems, with potential applications in biomedical engineering, microfluidic devices, and advanced thermal management systems.

Keywords: Walter's-B fluid; Bio-nanofluid; Heat transport; Gyrotactic microorganisms; Bioconvection; Non-Newtonian fluid; Nanomaterials.

I. INTRODUCTION:

The study of heat transfer in non-Newtonian fluids has gained significant attention due to its wide range of applications in industrial processing, biomedical engineering, polymer technology, and thermal management systems. Among various non-Newtonian models, Walter's-B fluid is particularly important for representing viscoelastic fluids with short memory effects, such as polymer solutions and biological fluids. Understanding heat transport behavior in such fluids is essential for optimizing processes where both fluid rheology and thermal performance play critical roles.

In recent years, the incorporation of nanoparticles into base fluids has emerged as an effective strategy to enhance thermal conductivity and heat transfer rates. These engineered suspensions, known as nanofluids, exhibit superior thermal characteristics compared to conventional fluids. However, challenges such as nanoparticle agglomeration and sedimentation can adversely affect stability and heat transfer efficiency. To overcome these issues, the concept of bio-convection induced by gyrotactic microorganisms has been introduced, providing a natural mechanism for improving nanoparticle dispersion and flow stability.

Gyrotactic microorganisms, which tend to swim upward due to a balance between gravitational and viscous torques, generate collective motion that induces bioconvection within the fluid. This phenomenon significantly alters the flow structure and enhances mixing, leading to improved heat and mass transfer characteristics. When combined with nanofluids and non-Newtonian fluid models, bioconvection offers a promising approach for controlling thermal transport in complex fluid systems.

Motivated by these developments, the present work focuses on the heat transport and bio-convective behavior of a Walter's-B nanofluid containing gyrotactic microorganisms. By formulating and analyzing a comprehensive mathematical model, this study aims to elucidate the influence of key physical parameters on flow dynamics and thermal performance. The findings contribute to the growing literature on bio-nanofluid dynamics and provide theoretical insights for applications in biomedical devices, microfluidic systems, and advanced heat transfer technologies.

II. MODELING

Here, the newly developed concept of Rosseland approximation and gyrotactic microorganisms in two-dimensional, steady, incompressible non-magnetized flow of Walter's-B fluid is addressed towards a stretched surface. The flow is nonlinear and generated by stretching phenomenon. The energy equation is based on the first law of thermodynamics and modeled in the presence of radiative heat flux and heat generation/absorption. Furthermore, concentration and motile density is discussed. Both thermal and solutal stratification conditions are imposed at the boundary of the stretched surface, which comprising of fluid parcels of various densities. Flow diagram is presented in Fig. 1.Fig. 2a.Fig. 2b.

Let $u = U_w(x) = ax$ highlights the stretching velocity of the sheet. The governing partial differential equations are considered flow problem are listed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_0}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{k_0}{\rho}\left(u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right). \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{(\rho c_p)_f}\left(\frac{\partial^2 T}{\partial y^2}\right) + \tau D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \frac{\tau D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 - \frac{1}{(\rho c_p)_f}\frac{\partial q_r}{\partial y} + \frac{Q_0}{(\rho c_p)_f}(T - T_\infty). \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} + \frac{bW_c}{(C_w - C_0)}\left[\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)\right] = D_m\frac{\partial^2 N}{\partial y^2}. \tag{5}$$

$$u = U_w(x) = ax, \ v = 0, \ T = T_w = T_0 + Ax, \ C = C_w = C_0 + Bx, \ N =$$
$$u \to 0, \ T \to T_\infty = T_0 + Ex, \ C \to C_\infty = C_0 + Fx, \ N \to N_\infty = ?$$

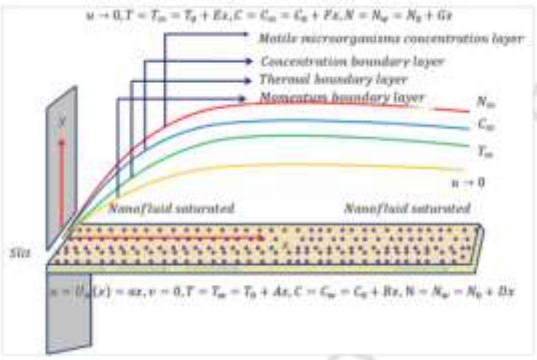

Fig. 1. Schematic flow diagram

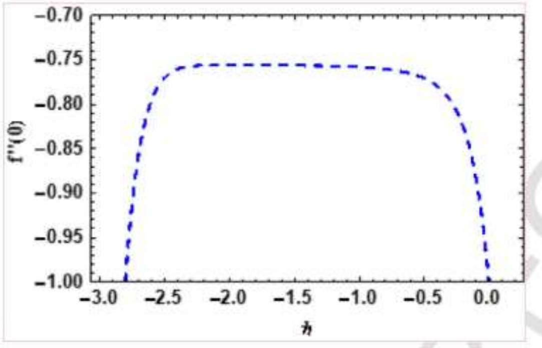

Fig. 2a. $\hbar - curve for f''(0)$.

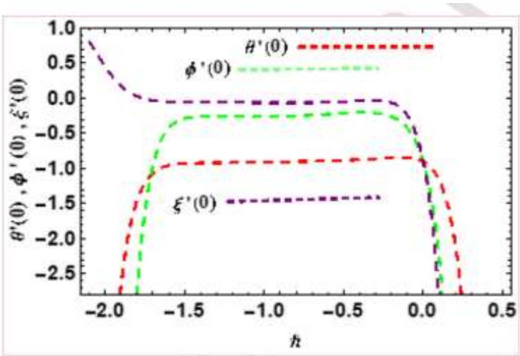

Fig. 2b. $\hbar - curve for \theta'(0), \phi'(0) and \xi'(0)$.

Note that, x,ysignifies the Cartesian coordinates, density, velocity components, short memory coefficient, viscosity, temperature, thermal conductivity, specific heat, ratio of heat $\tau\left(=\frac{(\rho c_p)_p}{(\rho c_p)_f}\right)$ capacities, Brownian motion, concentration, thermophoretic diffusion, coefficient of radiative heat flux, coefficient of heat generation/absorption, ambient temperature, concentration of microorganisms, ambient concentration, chemotaxis constant, microorganisms diffusion coefficient, maximum cell swimming velocity, stretching velocity, dimensional constant, wall temperature, dimensional constant, reference temperature, concentration, motile microorganisms and ambient motile density of microorganisms. The radiative heat flux in the presence Rosseland approximation is defined as mophoretic diffusion, coefficient of radiative heat flux, coefficient of heat generation/absorption, ambient temperature, concentration of microorganisms, ambient concentration, chemotaxis constant, microorganisms diffusion coefficient, maximum cell swimming velocity, stretching velocity, dimensional constant, wall temperature, dimensional constant, reference temperature, concentration, motile microorganisms and ambient motile density of microorganisms.

The radiative heat flux in the presence Rosseland approximation is defined as

$$q_r = -\frac{4\sigma^*}{k^*}\frac{\partial T^4}{\partial y} = -\frac{16\sigma^*}{3k^*}\frac{\partial T}{\partial y}, \qquad (7)$$

where signifies the Stefan-Boltzman constant and and mean absorption coefficient.

Letting

$$\left.\begin{array}{l} u = axf'(\eta), \quad v = -\sqrt{a\nu}f(\eta), \quad \eta = \sqrt{\frac{a}{\nu}}y, \\ \theta(\eta) = \frac{T-T_\infty}{T_w-T_0}, \quad \phi(\eta) = \frac{C-C_\infty}{C_w-C_0}, \quad \xi(\eta) = \frac{N-N_\infty}{N_w-N_0}, \end{array}\right\} \qquad (8)$$

We arrive

$$f''' + ff'' - f'^2 + \beta[f''^2 - 2f'f''' + ff^{(iv)}] = 0, \qquad (9)$$

$$\begin{array}{l} \theta'' + \frac{4}{3}R[3\delta\theta'^2 + 3\delta^2\theta^2\theta'^2 + 6\delta^2\theta\theta'^2 + \theta'' + \delta^3\theta^3\theta'' + 3\delta^2\theta^2\theta'' + 3\delta\theta\theta''] \\ \quad - PrS_1f' - Prf'\theta + Prf\theta' + Pr\delta_1\theta + PrNb\theta'\phi' + PrNt\theta'^2 = 0, \end{array} \qquad (10)$$

$$\phi'' + \frac{Nt}{Nb}\theta'' + Scf\phi' - Scf'\phi - ScS_2f' = 0, \qquad (11)$$

$$\xi''' - LbS_3 f' - Lbf'\xi + Lbf\xi' - Pe\left[\phi'\xi' + \Omega\phi'' + \xi\phi''\right], \quad (12)$$

$$f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1 - S_1,\ \phi(0) = 1 - S_2,\ \text{and}\ \xi(0) = 1 - S$$
$$f'(\infty) \to 0,\ \theta(\infty) \to 0,\ \phi(\infty) \to 0,\ \text{and}\ \xi(\infty) \to 0.$$

where $\beta\left(= \frac{\alpha k_0}{\mu_0}\right)$ highlights the Weissenberg number, $R\left(= \frac{4\sigma^* T_\infty^3}{3k^* k}\right)$ thermal radiation parameter, $\delta = \left(\frac{T_w - T_o}{T_\infty}\right)$ temperature ratio parameter,

$\Pr\left(= \frac{\mu_0 c_p}{k}\right)$ Prandtl number, $S_1\left(= \frac{E}{A}\right)$ thermal stratification parameter, $\delta_1\left(= \frac{Q_0}{(\rho c_p)_f a}\right)$ heat generation absorption parameter, $Nb\left(= \frac{(\rho c_p)_p D_B (C_w - C_0)}{(\rho c_p)_f \nu}\right)$ Brownian motion parameter, $S_2\left(= \frac{F}{B}\right)$ concentration stratification parameter, $Nt\left(= \frac{(\rho c_p)_p D_T (T_w - T_0)}{(\rho c_p)_f \nu T_\infty}\right)$ thermophoretic parameter, $Sc\left(= \frac{\nu}{D_B}\right)$ Schmidt number, $Lb\left(= \frac{\nu}{D_m}\right)$ Bio-convection Lewis number, $\Omega\left(= \frac{N_\infty}{N_w - N_0}\right)$ microorganisms concentration difference parameter, $Pe\left(= \frac{bW_c}{D_m}\right)$ Bio-convection Peclet number and $S_3\left(= \frac{G}{D}\right)$ motile density stratification parameter.

III. PHYSICAL INTEREST

In mathematical point of view, the skin friction coefficient, local density number, heat transfer rate and gradient of concentration are communicated as

$$\left.\begin{aligned} C_{fx} = \frac{2\tau_w}{\rho U_w^2},\ Nu_x = -\frac{xq_w}{k(T_w - T_0)}, \\ Sh_x = \frac{xq_m}{D_B(C_w - C_0)}\ \text{and}\ Nn_x = \frac{xq_n}{D_n(N_w - N_0)}, \end{aligned}\right\} \quad (14)$$

where and are addressed as

$$\left.\begin{aligned} \tau_w = \mu_0\left(\frac{\partial u}{\partial y}\right)_{y=0} - k_0\left(u\frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right),\ q_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0} \\ q_w = -k_f\left(1 + \frac{16\sigma^* T^3}{3k^* k_f}\right)\left(\frac{\partial T}{\partial y}\right)_{y=0},\ q_n = -D_n\left(\frac{\partial N}{\partial y}\right)_{y=0} \end{aligned}\right\} \quad (15)$$

The dimensionless form is

$$\left.\begin{aligned} \tfrac{1}{2} C_{fx} Re_x^{0.5} &= f''(0)\left[1 + \beta f'(0)\right], \\ Nu_x Re_x^{-0.5} &= -\left[1 + \tfrac{4}{3} R(1 + \delta\theta(0))^3\right]\theta'(0), \\ Sh_x Re_x^{-0.5} &= -\phi'(0),\ Nn_x Re_x^{-0.5} = -\xi'(0), \end{aligned}\right\} \quad (16)$$

where signifies the local Reynold number.

IV. METHODOLOGY

Here HAM is implemented to get the series solution of governing equations. The initial guesses and linear operators are communicated as

$$\left.\begin{aligned} f_0(\eta) = 1 - e^{-\eta},\ \theta_0(\eta) = (1 - S_1)\, e^{-\eta}, \\ \phi_0(\eta) = (1 - S_2)\, e^{-\eta},\ \xi_0(\eta) = (1 - S_3)\, e^{-\eta}. \end{aligned}\right\} \quad (17)$$

$$\left.\begin{aligned} \Im_f = f''' - f',\ \Im_\theta = \theta'' - \theta \\ \Im_\phi = \phi'' - \phi,\ \Im_\xi = \xi'' - \xi, \end{aligned}\right\} \quad (18)$$

which satisfies the characteristics

$$\left.\begin{aligned} \Im_f\left(c_1 + c_2 e^{\eta} + c_3 e^{-\eta}\right) = 0,\ \Im_\theta\left(c_4 e^{\eta} + c_5 e^{-\eta}\right) = 0, \\ \Im_\phi\left(c_6 e^{\eta} + c_7 e^{-\eta}\right) = 0,\ \Im_\xi\left(c_8 e^{\eta} + c_9 e^{-\eta}\right) = 0, \end{aligned}\right\} \quad (19)$$

where $c_i\ (i = 1 - 9)$ indicates arbitrary constants

V. CONVERGENCE ANALYSIS

In homotopy approach auxiliary variables i.e., and control and regulate region of convergence of series solution. In Fig. 2a and Fig. 2b we have plotted the curves for velocity, temperature, concentration and motile density profiles.

The suitable ranges of auxiliary variables i.e., $\hbar_f$, $\hbar_\theta$, $\hbar_\phi$ and $\hbar_\xi$ are $-2.8 \leqslant \hbar_f \leqslant 0.1$, $-2.0 \leqslant \hbar_\theta \leqslant 0.2$, $-1.8 \leqslant \hbar_\phi \leqslant 0.2$, $-1.6 \leqslant \hbar_\xi \leqslant 0.1$. The convergence of obtained solutions is also justified numerically in Table 1. From Table 1, it is noticed that 20th order of iterations are suff-

Table 1
Various numerical iterations for $f''(0)$, $\theta'(0)$, $\phi'(0)$ and $\xi'(0)$.

Iterations	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-\xi'(0)$
1	0.866	0.799	0.260	−0.022
5	0.772	0.878	0.189	0.038
10	0.759	0.903	0.257	0.064
15	0.757	0.916	0.258	0.058
20	0.756	0.926	0.261	0.060
25	0.756	0.927	0.264	0.060
30	0.756	0.927	0.264	0.060

cient for $f''(0)$ and $\xi'(0)$ and 25th order of iterations are sufficient for $\theta'(0)$ and $\phi'(0)$. Table 2 represents the comparative investigation of present work with Saleem el al. [54] and noticed a very good agreement with them.

Discussion

Salient characteristics of pertinent flow parameters on the $(f'(\eta))$, $(\theta(\eta))$, $(\phi(\eta))$ and $(\xi(\eta))$ areanalyzed in this section, where indicates the velocity field, $\theta(\eta)$highlights the temperature, $(\phi(\eta))$ signifies the concentration and denotes the motile density.

VI. CONCLUSIONS

This study presents a detailed analysis of heat transport in the bio-convective flow of a Walter's-B non-Newtonian nanofluid containing gyrotactic microorganisms. The combined effects of fluid viscoelasticity, nanoparticle dynamics, and microorganism-induced bioconvection were examined to understand their influence on flow behavior and thermal performance. The results demonstrate that the presence of gyrotactic microorganisms significantly enhances fluid mixing and stabilizes nanoparticle distribution, leading to improved heat transfer characteristics.

It was observed that key parameters such as the viscoelastic parameter, Brownian motion, thermophoresis, and bioconvection Rayleigh number play crucial roles in governing velocity, temperature, and microorganism density profiles. Increased bioconvection intensity was found to augment thermal transport by reducing nanoparticle sedimentation and promoting uniform temperature distribution. Overall, the findings highlight the effectiveness of coupling nanofluids with bioconvection in Walter's-B fluids to achieve enhanced heat transfer. The study offers valuable theoretical insights for the design and optimization of advanced thermal systems in biomedical engineering, microfluidics, and energy-related applications.

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